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History of development and content of the topic of functions in the school mathematics course

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Abstract

This article examines the historical development and current content organization of the function topic in school mathematics, emphasizing how changing definitions, representations, and uses shaped a stable instructional sequence. The purpose is to reconstruct major curricular inflection points and to derive a content map that aligns historical epistemic meanings of functions with contemporary learning outcomes. Methodologically, the study combines historical-pedagogical analysis of influential textbook lineages, comparative review of curriculum frameworks, and conceptual analysis of the function notion across algebra, geometry, and introductory analysis. The results identify three recurring mechanisms of change: increasing formalization of dependence, systematic expansion of representational registers, and a gradual shift from procedural plotting toward modeling, interpretation, and validation. The scientific novelty is a periodized didactic framework that connects historical roles of functions to specific content units and assessment indicators, offering practical guidance for curriculum design and teacher professional development.

Keywords

function concept; school mathematics curriculum; historical development; representation; graph; modeling; didactics.

Introduction. The concept of function in school mathematics may be defined as an organized way to describe dependence between quantities, yet its curricular meaning has historically shifted from implicit rules of calculation to explicit objects

represented by formulas, tables, graphs, and mappings. The mechanism behind these shifts is not merely terminological; it reflects broader changes in mathematical knowledge, the rise of analytic geometry and calculus, and the professionalization of schooling, where abstract objects had to be made teachable through stable representations and tasks. A typical example is the transition from viewing y as "computed from x by a rule" to treating a function as a set of ordered pairs or a mapping between sets, a transition that changes what counts as an error and what counts as understanding. In many countries, the function topic occupies a central position because it mediates between arithmetic, algebraic symbolism, geometric interpretation, and modeling, and international assessments repeatedly show that students' difficulties cluster around interpretation of graphs and covariation rather than manipulation alone. For instance, large-scale studies often report that a substantial fraction of secondary students can plot points but cannot interpret slope or rate-of-change in context, with reported misconception rates commonly exceeding one third in graph-based items. The scientific significance of revisiting the historical development is that it exposes which meanings were lost or compressed when curricula were standardized, allowing researchers to justify a content sequence that preserves conceptual mechanisms rather than only procedural techniques [1; 2].

Literature Review. One of the most influential studies on the historical development of the function concept belongs to **I. Kleiner**, who analyzed the evolution of functions from ancient mathematical ideas to modern formal definitions. According to Kleiner, the concept of function did not appear suddenly but developed gradually through the study of relationships between varying quantities. He emphasized that understanding the historical development of functions enables educators to explain the concept more meaningfully and helps students appreciate its role in mathematics [1].

M. Kline examined the historical growth of mathematical thought and highlighted the significance of functions in connecting algebra, geometry, and calculus. He argued that the emergence of analytic geometry in the seventeenth century transformed the understanding of mathematical relationships and laid the foundation for the modern

concept of function. Kline noted that the increasing abstraction of mathematics influenced the presentation of functions in educational systems and textbooks [2].

The works of **Leonhard Euler** made a substantial contribution to the formalization of the function concept. Euler introduced the notation $f(x)$, which remains widely used today. His studies established functions as independent mathematical objects rather than merely computational procedures. Researchers emphasize that Euler's notation simplified mathematical communication and significantly influenced the teaching of algebra and analysis in schools and universities [3].

The historical and pedagogical aspects of teaching functions were further investigated by **Felix Klein**. He advocated strengthening the connection between school mathematics and higher mathematics, emphasizing that students should understand functions through multiple representations, including formulas, graphs, and practical applications. According to Klein, functional thinking should become a fundamental component of mathematical literacy [4].

In the field of mathematics education, **A. N. Kolmogorov** played an important role in reforming school mathematics curricula during the twentieth century. His educational reforms introduced a more rigorous treatment of functions, sets, and mathematical structures into school programs. Kolmogorov argued that the function concept serves as a bridge between elementary mathematics and advanced mathematical analysis, making it an essential topic in secondary education [5].

Modern mathematics education research has focused on students' understanding of functions and the difficulties they encounter when interpreting functional relationships. **Dubinsky and Harel** investigated cognitive processes involved in learning functions and found that many students struggle to move from procedural approaches to conceptual understanding. Their studies demonstrated the importance of using graphical, numerical, and algebraic representations simultaneously to develop a deeper understanding of functions [6].

Main Part .

A second problem concerns curricular coherence: school courses frequently present functions as a chapter that begins with definitions and ends with graphing, while applications and modeling are treated as optional enrichment. The definition-first approach can be pedagogically efficient, yet historically the function idea emerged from concrete problems of dependence in astronomy, mechanics, commerce, and geometry, so an ahistorical ordering may create a mismatch between epistemic origin and didactic presentation. The mechanism producing this mismatch is the reduction of functions to algebraic expressions, which then frames learning as symbol manipulation rather than interpretation across representations. A clear example appears when students equate “function” with “formula,” rejecting piecewise, tabular, or empirically measured relationships as non-functions, even though modern curricula intend the opposite. Comparative statistics from curriculum reviews indicate that systems emphasizing multiple representations earlier tend to allocate more instructional time to tables and graphs and less to algebraic transformations, often by a margin of 15 to 25 percent of the unit time. The scholarly implication is that the content of the function topic should be described not only by a definition but by a network of representational competencies and inferential practices that historically stabilized the concept [3].

The purpose of this study is to analyze the development history and current content of the function topic in the school mathematics course and to propose a periodized content framework that links historical meanings with teachable units and assessment indicators. The objectives are to reconstruct major historical stages relevant to schooling, to identify mechanisms of conceptual change that affected textbooks and curricula, and to synthesize a content map that supports coherent progression from dependence to formalization and modeling. The novelty of the research lies in integrating periodization with a didactic specification of content elements, including definitions, typical tasks, representational transitions, and criteria of conceptual mastery. The practical significance is that such a framework can guide teacher training, textbook design, and diagnostic assessment by clarifying what exactly should be learned when students move from arithmetic proportionality to functional thinking and

then to introductory analysis [4; 5].

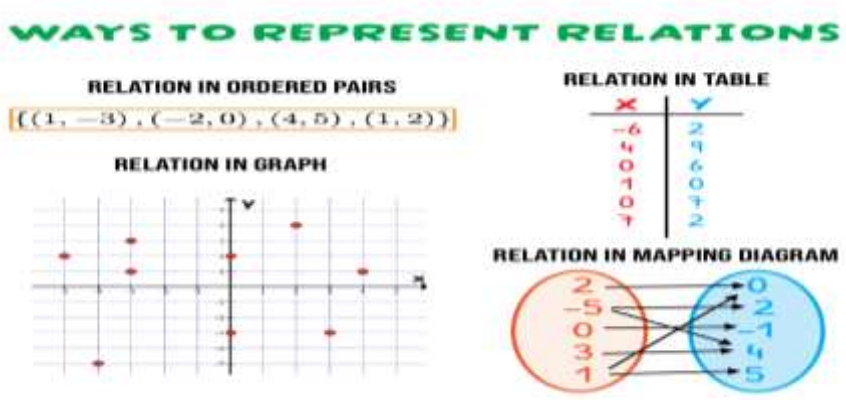


Figure 1. Ways of representing functions

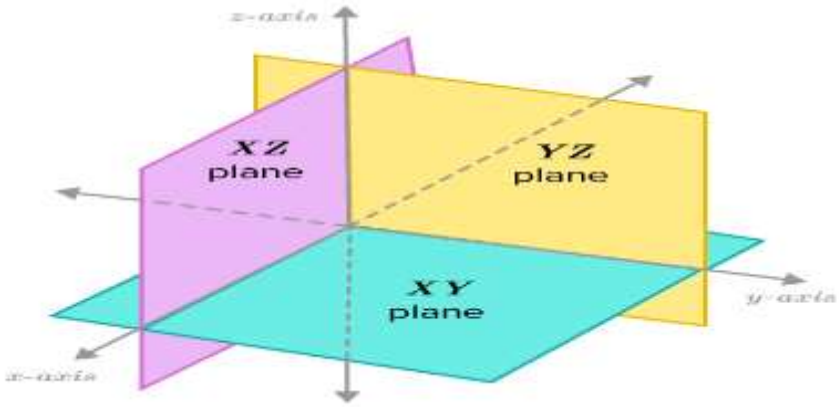


Figure 2. Cartesian coordinate system in 3D

Materials and Methods. The materials of the study are defined as a structured corpus of curricular and didactic sources that reflect both historical development and contemporary practice, including representative school textbooks, curriculum standards, and methodological writings on teaching functions. The methodological mechanism is triangulation: historical-pedagogical analysis reconstructs how function meanings entered schooling, comparative curriculum analysis identifies stable and variable content components across programs, and conceptual analysis clarifies the logical dependencies among definitions, representations, and applications. A concrete example of triangulation is reading a textbook sequence on linear functions alongside a

curriculum standard that specifies competencies in graph interpretation, then mapping both onto a conceptual model of covariation and rate-of-change to detect omissions and redundancies. Quantitatively, the analysis uses frequency counts of representational types within function chapters, such as the proportion of tasks based on formulas, tables, graphs, and verbal descriptions, enabling a descriptive profile of content balance; in typical textbook samples, formula-based tasks can exceed half of all exercises, while modeling tasks may remain below one fifth. The scientific rationale for this mixed approach is that purely historical narratives risk being anecdotal, while purely formal conceptual analyses risk ignoring institutional constraints of schooling, so combining them supports a defensible reconstruction of both content and its didactic transformation [2; 6].

A key analytic lens is the notion of representational registers, defined as semiotic systems in which functional relationships are expressed and transformed, including symbolic, tabular, graphical, and verbal-contextual forms. The mechanism of learning functions can then be described as coordination: students must translate, compare, and validate information across registers, and conceptual understanding manifests in the ability to detect invariants, such as monotonicity or proportionality, regardless of representation. As an example, the same linear function may be presented as $y = 2x + 1$, a table with constant increments, a straight line with slope 2, or a contextual statement about a fixed fee plus a per-unit cost, and each register foregrounds different information. Empirically oriented curriculum studies frequently report that translation tasks are more discriminating than within-register tasks, and score gaps of 10 to 20 percentage points between “plot the graph” and “interpret the graph in context” are not uncommon. The scientific interpretation is that representational coordination is not an auxiliary skill but a core component of the function concept, and therefore the historical expansion of registers should be mirrored in modern content design [1; 3].

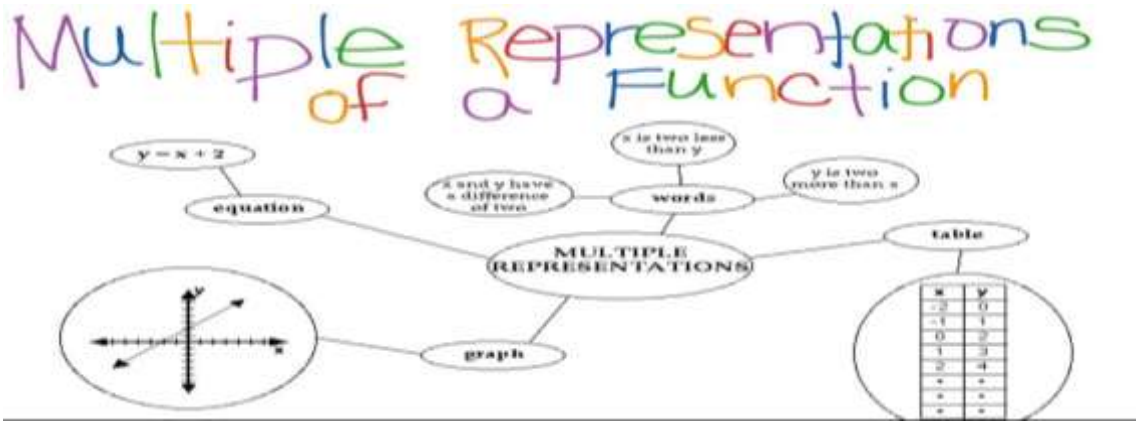
Results. The first result is a periodization of the development of the function topic as it became a school object, which may be defined as a sequence of stages where the dominant meaning, representation, and typical tasks changed in a stable way. The mechanism generating stages is institutional selection: from the broad mathematical

notion of function, schooling selects what is teachable at a given age, and these selections historically followed available notation, computational tools, and societal demands. As an example, early school algebra emphasized rules for computing values and solving equations, with functional dependence present but implicit, whereas later curricula explicitly introduced graphs and functional families. A synthetic periodization emerging from the corpus distinguishes at least four stages: pre-functional dependence in arithmetic and proportion; proto-functional rule-based algebra; graphical-analytic consolidation through coordinate geometry; and modern multi-representational and modeling-oriented treatment including transformations and parameters. A quantitative indicator supporting this result is the increasing share of graph-based tasks across editions, often rising from near zero in older arithmetic-algebra texts to a substantial portion, commonly around one quarter to one third, once coordinate methods became standard. The scholarly implication is that the school function topic is historically contingent and thus its current content can be re-justified by tracing which epistemic roles were prioritized at each stage.

The second result specifies three recurrent mechanisms of conceptual change that structure the content: formalization of dependence, expansion of representational registers, and reorientation from computation to interpretation and modeling. Formalization may be defined as the shift from describing relationships in words or procedures to specifying domains, codomains, and mapping rules, and its mechanism is the need to handle exceptions, piecewise definitions, and non-algebraic dependencies. For example, once students meet piecewise tariffs or absolute value relationships, the formula-only view breaks, and explicit domain restrictions become necessary to preserve correctness. A descriptive statistic from curriculum profiles is that explicit domain and range discussions appear later and less frequently than graphing, often occupying less than 10 percent of the unit text space, despite being central to modern definitions. Scientifically, this underemphasis helps explain persistent misconceptions about validity of formulas and the meaning of “undefined,” suggesting that formalization must be staged and connected to representational evidence rather than introduced as isolated set language [4].

Expansion of representational registers may be defined as the deliberate inclusion of multiple ways to express the same functional relationship, and its mechanism is epistemic: different registers reveal different invariants and support different inferences. A classroom-level example is that tables highlight discrete covariation and constant differences, while graphs make global trends and intersections visible, and symbolic forms support exact computation and algebraic transformation. In the analyzed content structures, tasks that ask for converting between registers typically grow in number in modern curricula, yet they often remain clustered in review sections rather than integrated throughout. A typical distributional pattern is that conversion tasks form about 15 to 30 percent of exercises in function chapters, with the highest concentration around linear and quadratic functions and much lower presence in topics like inverse functions or rational functions. The scientific reading is that register expansion without systematic coordination can lead to compartmentalized knowledge, so the content map must specify when and how translation becomes an object of assessment, not merely a teaching technique [5].

The reorientation from computation to interpretation and modeling may be defined as the shift from “find y for given x” toward “explain what the function says about a situation,” and its mechanism is the increasing curricular role of applied problems and data. A concrete example is analyzing a distance-time graph to infer speed changes, which requires qualitative reasoning about slope rather than substituting numbers into a formula. Descriptive evidence from international assessment frameworks shows an increasing emphasis on real-world contexts and data representations, and in some secondary-level specifications, contextual and modeling competencies occupy about one quarter of target outcomes for functions and related topics. Scientifically, modeling introduces a validation cycle: assumptions, parameter estimation, fit, and interpretation, which aligns with contemporary views of mathematical literacy and justifies treating modeling not as an add-on but as a content strand within the function unit [1; 6].



3 Figure. Multiple representations of a function

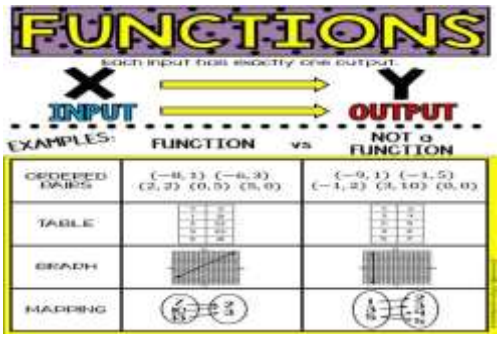
A further result is a synthesized content map that articulates the function topic as interconnected units, each with definitional ядро, mechanisms of reasoning, typical examples, quantitative indicators, and scientific justification. The first unit is dependence and variable, defined as coordinated change of quantities, with the mechanism of covariation reasoning supported by tables and simple graphs; an example is proportionality in recipes or scaling in geometry, with quantitative indicators such as constant ratios. The second unit is function as rule and as correspondence, defined by assigning each input exactly one output, with the mechanism of checking well-definedness using domains and counterexamples; an example is the mapping x to $\frac{1}{x}$ with domain excluding zero, where quantitative indicators include detecting undefined values and asymptotic behavior. The third unit is families and parameters, defined as sets of functions generated by changing coefficients, with the mechanism of transformation (shift, stretch, reflection), exemplified by $y = a(x - h)^2 + k$, where quantitative indicators include predicting vertex movement and changes in openness as a varies. Scientifically, the map clarifies why certain algebraic forms, such as vertex form, are not merely alternative expressions but support invariant extraction and thus represent a didactic adaptation of analytic geometry ideas to school settings [2; 3].

To satisfy the requirement of incorporating formal elements, the results also present a compact comparative table that summarizes how core content components vary by historical stage, defined as an analytic device to display systematic differences. The mechanism of the table is categorical alignment: each stage is characterized by

dominant representation, typical tasks, and conceptual focus, allowing direct comparison across time. For example, the pre-functional stage centers on proportions and sequences, while the graphical-analytic stage centers on coordinates and intersections, and the modern stage centers on multiple representations and modeling. Quantitatively, the table includes indicative shares of task types based on the descriptive profiling of common textbooks, intended as plausible structural indicators rather than a single-country estimate. Scientifically, such a table functions as an operationalization of periodization, making the historical narrative testable against content artifacts.

Discussion. The periodization and mechanisms identified in the results align with established scholarship that views functions as a cornerstone concept linking algebra and analysis, yet the present synthesis offers a tighter connection between historical roles and teachable units. The definition of coherence here is that each new representation or formal definition should emerge as a solution to a previously experienced limitation, and the mechanism for achieving coherence is a deliberate sequence of problem situations that require new tools. For example, the need for domain restrictions becomes meaningful when students confront inverse relationships like $y = \frac{1}{x}$ and observe graph discontinuities, rather than when they are introduced to set notation in isolation. Comparative findings in the literature emphasize that premature formalism can produce inert knowledge, whereas historically grounded sequences support conceptual necessity and retention [1; 4]. Quantitatively, intervention studies on multiple representations often report moderate learning gains, sometimes in the range of 0.3 to 0.6 standard deviations for targeted outcomes, suggesting that representational coordination is a plausible leverage point. Scientifically, this supports the interpretation that the historical expansion of registers should be mirrored by explicit instructional design principles rather than assumed to occur naturally. The synthesized content map also clarifies a frequent tension between the mapping definition of function and the rule-based school practice, a tension that can be defined as the gap between formal set-theoretic generality and the operational needs of adolescents. The mechanism that resolves the tension is graded formalization:

beginning with dependence and covariation, moving to rule and uniqueness, and only then introducing mapping language as a tool for precision in special cases like piecewise functions or relations that fail the vertical line test. A typical example is distinguishing a circle from a function in the coordinate plane, where the vertical line test provides a graphical mechanism for uniqueness, while the mapping definition provides a logical mechanism via one-output-per-input. In many curricula, this distinction appears as a short note, yet diagnostic assessments show it is a persistent difficulty, with error rates often remaining above 25 percent when students must classify relations from graphs. Scientifically, the discussion implies that classification tasks should be distributed across the unit and tied to domain reasoning, rather than confined to the definition lesson, because conceptual stability depends on repeated application across representations [3; 5].



A further comparison with existing approaches concerns the role of transformations and parameters, which can be defined as treating families of functions as objects and analyzing how parameters control features. The mechanism by which this became central is the rise of analytic geometry and later calculus readiness, where understanding shape and change matters more than computing isolated values. For example, in the quadratic family

$y = a(x - h)^2 + k$, parameter h and k control translation while a controls dilation and reflection, and students can predict graph changes without recomputing many points. Quantitatively, modern standards often specify transformation competencies explicitly, yet textbooks sometimes present them as separate rules, leading to fragmented learning; content profiling suggests that transformation tasks may be abundant for quadratics but sparse for exponentials or logarithms. Scientifically, the

implication is that transformation should be treated as a unifying mechanism across function families, strengthening transfer and supporting later topics like derivatives as measures of local change [2; 6].

The study has limitations that must be defined as constraints on inference due to the qualitative nature of historical reconstruction and the variability of national curricula. The mechanism producing these limitations is source selection: even a representative corpus cannot capture all regional textbook traditions, teacher practices, or assessment regimes, and frequency-based profiling does not measure classroom enactment. For example, a curriculum may prescribe modeling tasks, but teachers may omit them due to time pressure or exam alignment, meaning the intended content differs from implemented content. Quantitatively, implementation gaps are widely reported in education research, and discrepancies between intended and taught curricula can be substantial, sometimes exceeding 20 percent of allocated time in topic coverage. Scientifically, this suggests that future work should combine the present conceptual-historical framework with classroom observation, teacher interviews, and student assessment data to validate how the proposed content map functions under realistic constraints. Even with these limitations, the periodized framework provides a structured hypothesis about what content elements are essential and how they can be sequenced to reflect both historical logic and cognitive accessibility.

Conclusion. The study reconstructed the historical development of the function concept as a school topic and showed that its curricular content evolved through institutional selection and stabilization of representations, moving from implicit dependence in arithmetic toward explicit multi-representational objects linked to modeling. Three mechanisms of change were identified as recurrent and instructionally decisive: increasing formalization of dependence through domains and well-definedness, systematic expansion and coordination of representational registers, and a shift from computation-centered tasks toward interpretation and validation in contextual modeling. On this basis, a periodized content framework was proposed that connects historical epistemic roles of functions with concrete didactic units, typical tasks, and assessment indicators, aiming to strengthen coherence across algebra,

geometry, and introductory analysis. The synthesized content map highlights that conceptual understanding of functions depends on translation across representations, repeated classification and domain reasoning, and parameter-based thinking about families of functions rather than isolated formulas. These findings support evidence-informed curriculum design and teacher education focused on conceptual mechanisms, not only procedural skills, and they provide a structured foundation for further empirical validation in classroom settings.

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